Security-Constrained MIP formulation of Topology Control Using Loss-Adjusted Shift Factors

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***Abstract*—Following up on our previous work, we formulate a linear, loss-adjusted, shift factor mixed integer program (MIP) to co-optimize generation and network topology. While both the $B\theta$ and shift factor topology control (TC) formulations lead to production cost saving, we showed that the shift factor formulation performs better for small to medium switchable sets. In this paper we extend the original TC shift factor formulation to include marginal losses. We derive loss-adjusted shift factors and show that both losses and flows can be updated linearly with a change in topology by taking advantage of flow-canceling transactions (FCTs). The marginal loss formulation we present in this paper closely resembles that of most market engines. In doing so, we aim to better approximate the AC power flow and to generate topologies leading to production cost savings while maintaining feasibility subject to AC security constrained power flow (SCOPF) constraints.**

I. Nomenclature

Vectors are indicated by lower case bold, matrices by upper case bold, and scalars by lower case italic characters indexed appropriately. Upper limits are indicated by an over-bar, and lower limits by an under-bar. Diagonal matrices are denoted with a tilde. Sensitivities are indicated with Greek characters.

Indices

m, n	Nodes.
k, ℓ	Lines.
m_ℓ	Line ℓ *from* node.
n_ℓ	Line ℓ *to* node.
τ	Contingent topologies.

Contingent Topology-Dependent Parameters and Variables

For contingent topology τ,

$\mathbf{f}_\tau$	Vector of real power flows on transmission elements.
$\mathbf{g}_\tau$	Bias from linearization of transmission flows.
$\underline{\mathbf{f}}_\tau, \overline{\mathbf{f}}_\tau$	Vectors of transmission limits.
$\underline{\tilde{\mathbf{F}}}_\tau, \overline{\tilde{\mathbf{F}}}_\tau$	Diagonal matrices of transmission limits.
$\mathbf{v}_\tau$	Vector of flow-canceling transactions.
$\mathbf{\Psi}_\tau$	Shift factor matrix.
$\hat{\mathbf{\Psi}}_\tau$	Loss-adjusted shift factor matrix.
$\mathbf{\Psi}_\tau^{\mathcal{M}}$	Shift factor matrix associated with monitored lines.
$\mathbf{\Psi}_\tau^{\mathcal{S}}$	Shift factor matrix associated with switchable lines.
$\mathbf{\Phi}_\tau^{\mathcal{SS}}$	PTDF matrix of switchable lines for transfer between switchable line terminals.
$\mathbf{\Phi}_\tau^{\mathcal{MS}}$	PTDF matrix of monitored lines for transfer between switchable line terminals.
$\hat{\mathbf{\Phi}}_\tau$	Loss-adjusted PTDF matrix.
$\psi_{\ell\tau}^m$	Element of $\mathbf{\Psi}$ for line ℓ, node m.
$\phi_{\ell\tau}^k$	PTDF of line ℓ for a transfer across line k.
$o_{\ell\tau}^k$	LODF of line ℓ for the outage of line k.
$\underline{\boldsymbol{\mu}}_\tau, \overline{\boldsymbol{\mu}}_\tau$	Monitored facilities shadow prices.
$\underline{\boldsymbol{\alpha}}_\tau, \overline{\boldsymbol{\alpha}}_\tau$	Switchable facilities shadow prices.

Contingent Topology-Independent Parameters and Variables

$\mathbf{1}$	Vector of ones.
$\mathbf{0}$	Vector of zeros.
$\mathbf{I}$	Identity matrix.
$\mathbf{z}$	Vector with the state of transmission lines.
$\mathbf{c}$	Vector of nodal generation variable cost.
$\mathbf{p}$	Vector of nodal generation.
$\mathbf{l}$	Vector of nodal loads.
$\boldsymbol{x}$	Line loss factors.
I_ℓ	Line current.
R_ℓ	Line resistance (RMS value).
V_ℓ	Nominal Line voltage (RMS value).
$\mathbf{d}$	Normalized vector that allocates total transmission losses to busses.
τ_0	Full topology without contingencies.
$\mathbf{f}^0$	Reference flows from topology τ_0 of a load flow solution.
$\boldsymbol{x}^0$	Line loss factors for a reference flow vector $\mathbf{f}^0$.
$\mathbf{b}^0$	Bias factor from linearization of transmission losses in topology τ_0.
$\mathbf{g}^0$	Bias factor from linearization of transmission flows in topology τ_0.
$\mathbf{l}$	Vector of nodal loads.
λ	Power balance shadow price.
η	Loss equation shadow price.
$\mathbf{\Psi}^{\mathcal{S}}$	Matrix of all $\mathbf{\Psi}_\tau^{\mathcal{S}}$.
$\mathbf{\Psi}^{\mathcal{M}}$	Matrix of all $\mathbf{\Psi}_\tau^{\mathcal{M}}$.
$\mathbf{O}^{\mathcal{MS}}$	LODF matrix of monitored branches for the outage of switchable branches.
M	Sufficiently large number.

Introduction

In recent years there has been a significant interest in co-optimization of transmission and generation in power system operation. Applications have varied from corrective control [1]–[3] to security enhancements [4], [5], loss minimization [6], [7] and more recently to production cost savings under economic dispatch [8]–[10] and unit commitment (UC) [11], [12]. Previous work has shown that even under a full security-constrained OPF, significant production cost savings can be achieved. In [13] we introduced a novel lossless, linear MIP formulation to efficiently model topology control (TC) using flow-canceling transactions (FCTs). This *shift factor TC formulation* is compact and scalable, especially when the set of candidate switchable lines is small relative to the number of monitored transmission elements. Relative to previous $B\theta$ implementations of (TC), the size of the shift factor formulation is a function of (the number of contingencies) times (the number of monitored and switchable transmission elements) as opposed to (number of contingencies) times (all transmission elements), which can be significant for large system. Additionally, the use of FCTs keeps the linear structure of the MIP and avoids re-calculation of the shift factor matrix with changes in transmission topology. In [13] we compare the computational performance of the FCT-based formulation to the previously used B-θ formulation and find that the FCT-based formulations result in significantly smaller solution times. Although the MIP formulation is linear, we observe exponential growth in computational time. For the IEEE 118-bus test case, a full $N-1$ security constrained (SC) OPF with more than 24 switchable lines becomes impractical to solve. To address this issue, we tested various heuristics such as price difference, line profit and total cost derivative[1] to identify candidate lines for switching and showed that through a combination of these heuristics we can achieve similar cost savings to the MIP while maintaining tractable solution times. Further, consistent with previous publications, most of the savings are realized with only a few line openings and these initial savings are almost fully captured by appropriate use of heuristics. We again note that the FCT-based approach avoids the need to recalculate the shift factor matrix with each topology change and is more efficient than the B-θ formulation in simulating these heuristics.

While the lossless DC formulation shows promising results, most modern power markets include a linearized model of losses in their market clearing algorithm. Therefore, for Independent System Operators(ISOs) to adopt topology control, it is important to incorporate marginal losses into the TC MIP formulation. Additionally, while TC provides benefits under the DC SCOPF context it must ultimately satisfy AC OPF constraints. If the DC optimized topology is not feasible with respect to AC constraints, it is often time consuming to restore feasibility while maintaining production cost savings. Including losses in the TC MIP formulation should lead to a closer approximation of the AC OPF and thus reduce the occurrence of AC-infeasible solutions.

The rest of this paper has six sections. Section II summarizes flow-canceling transactions and the lossless TC MIP formulation. Section III introduces a formulation with losses and Section IV formulates the TC MIP with losses. Section V discusses LMP decompositions with losses and Section VI compares the performance between formulations with and without losses and to the full AC OPF. Finally, Section VII concludes.

[1]See [14], [15] for details.

II. FCTs and the Lossless Shift Factor TC MIP

To motivate the importance of FCTs we first show the difficulty arising from a naïve shift factor implementation of TC by deriving the flow constraints. Consider first a transaction (pair of injection/withdrawal) of v MW from m_ℓ to n_ℓ. The change in flow on line k due to this transaction can be expressed using the power transfer distribution factor (PTDF) [16] as

$$\Delta f_k = \phi_k^\ell v = (\psi_k^{m_\ell} - \psi_k^{n_\ell})v$$

The change in flow on line k per unit of flow on line ℓ after line ℓ is disconnected is called the line outage distribution factor (LODF) [16] and is defined as

$$\begin{aligned} o_\ell^\ell &= -1 \\ o_k^\ell &= \frac{\phi_k^\ell}{1-\phi_\ell^\ell}, \ell \neq k, \phi_l^\ell \neq 1 \end{aligned}$$

Hence, after line ℓ is disconnected the change in flow on line k is

$$\Delta f_k^{(-\ell)} = \frac{\psi_k^{m_\ell} - \psi_k^{n_\ell}}{1-(\psi_\ell^{m_\ell} - \psi_\ell^{n_\ell})} f_\ell \tag{1}$$

where the superscript $(-\ell)$ denotes the disconnection of line ℓ. Using the definition of flow as $\mathbf{f} = \boldsymbol{\Psi}(\mathbf{p}-\mathbf{l})$ and (1) we have

$$\begin{aligned} &f_k^{(-\ell)} = f_k + \Delta f_k^{(-\ell)} = \\ &\boldsymbol{\psi}_k(\mathbf{p}-\mathbf{l}) + \frac{\psi_k^{m_\ell} - \psi_k^{n_\ell}}{1-(\psi_\ell^{m_\ell} - \psi_\ell^{n_\ell})}\boldsymbol{\psi}_\ell(\mathbf{p}-\mathbf{l}) \end{aligned} \tag{2}$$

where $\boldsymbol{\psi}_k$ denotes the row of $\boldsymbol{\Psi}$ corresponding to line k. Using our notation of $z_{ell} = 1$ to denote that line ℓ is closed and $z_\ell = 0$ to denote that it is disconnected, we can express the change in $\boldsymbol{\psi}_k$ conditional on the opening of line ℓ using (2) as

$$\Delta\boldsymbol{\psi}_k|z_\ell = \frac{\psi_k^{m_\ell} - \psi_k^{n_\ell}}{1-(\psi_\ell^{m_\ell} - \psi_\ell^{n_\ell})}\boldsymbol{\psi}_\ell(1-z_\ell)$$

We can see that using this shift factor in an OPF formulation would make the problem non-linear since $[\Delta\boldsymbol{\psi}_k|z_\ell](p-l)$ is not linear in the decision variables, z, p. The key behind the formulation developed in [13] is that it allows us to use FCTs to solve a linear MIP under the shift factor formulation. FCTs are a common tool for deriving line outage distribution factors [16] and are defined as pairs of injections and withdrawals at the end of lines to be opened that have the same impact on all flows in the rest of the system as actually opening these lines. If we open a line ℓ the impact this opening will have on the flow on any other line k is

$$\Delta f_k = o_k^\ell f_\ell \tag{3}$$

Alternatively, if we want to replicate this impact by using a FCT, the change in flow on line k due to an injection/withdrawal v_ℓ across line ℓ is

$$\Delta f_k = \phi_k^\ell v_\ell \tag{4}$$

Equating (3) and (4) gives

$$o_k^\ell f_\ell = \phi_k^\ell v_\ell \rightarrow \frac{\phi_k^\ell}{1-\phi_\ell^\ell} f_\ell = \phi_k^\ell v_\ell \rightarrow$$
$$v_\ell = \frac{f_\ell}{1-\phi_\ell^\ell} \tag{5}$$

exactly as shown in [13].

The resulting SCOPF formulation with TC is

$$\mathcal{C} = \min_{\mathbf{p},\mathbf{v},\mathbf{z}} \mathbf{c}'\mathbf{p} \tag{6}$$
$$\text{s.t.} \quad \mathbf{1}'(\mathbf{p}-\mathbf{l}) = 0, \tag{7}$$
$$\underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}}, \tag{8}$$
$$\underline{\mathbf{f}}_\tau^{\mathcal{M}} \leq \mathbf{\Psi}_\tau^{\mathcal{M}}(\mathbf{p}-\mathbf{l}) + \mathbf{\Phi}_\tau^{\mathcal{MS}}\mathbf{v}_\tau \leq \overline{\mathbf{f}}_\tau^{\mathcal{M}}, \quad \forall\tau \tag{9}$$
$$\underline{\tilde{\mathbf{F}}}_\tau^{\mathcal{S}}\mathbf{z} \leq \mathbf{\Psi}_\tau^{\mathcal{S}}(\mathbf{p}-\mathbf{l}) + \left(\mathbf{\Phi}_\tau^{\mathcal{SS}} - \mathbf{I}\right)\mathbf{v}_\tau \leq \overline{\tilde{\mathbf{F}}}_\tau^{\mathcal{S}}\mathbf{z}, \quad \forall\tau \tag{10}$$
$$-M(\mathbf{1}-\mathbf{z}) \leq \mathbf{v}_\tau \leq M(\mathbf{1}-\mathbf{z}), \quad \forall\tau \tag{11}$$
$$z_\ell \in \{0,1\}, \quad \forall\ell \tag{12}$$

We refer to problem (6)-(12) as the lossless shift factor TC formulation. The set of switchable lines $\mathcal{S}$ denotes the set of lines that are enforced and are candidates for switching while the set of monitored lines $\mathcal{M}$ denotes the set of lines that are enforced and may not be switched. For each opened line $\ell \in \mathcal{S}$, $z_\ell = 0$ and the corresponding FCT is unrestricted. For all such lines, constraints (10) become a set of equality constraints forcing the flow between opened lines and the rest of the system to zero, thus defining a simultaneous system of linear equations for the corresponding FCTs, which fall out directly from the linear formulation. Solving this system is the same as applying the principle of superposition [17] to[2] (5):

$$\mathbf{v}_\tau^{\mathcal{S}} = \mathbf{f}_\tau^{\mathcal{S}}\left(\mathbf{I} - \mathbf{\Phi}_\tau^{\mathcal{SS}}\right)^{-1} \tag{13}$$

For a single line ℓ, (13) is identical to (5) and substituting this value of v into equation (9) impacts flows on monitored lines exactly as in (2). Hence, FCTs give us a way to linearly replicate an update to the shift factor matrix while co-optimizing transmission and generation.

III. Linearized Loss Model Formulation

Resistive losses are a quadratic function of current flowing on each transmission line:

$$Loss = \sum_k I_k^2 R_k = \sum_k \frac{f_k^2 R_k}{\cos^2\varphi_k V_k^2} \approx \sum_k \frac{f_k^2 R_k}{V_k^2}, \tag{14}$$

where φ_k is the angle difference between voltage and current and the approximation in the last equality depends on the assumptions that reactive power flows can be ignored (voltage and current are in phase, $\varphi_k = 0$). To incorporate a linear approximation of losses into the DC OPF we perform the standard Taylor series expansion around a base flow $\mathbf{f}^0$:

$$Loss \approx \mathbf{b}^0 + \left.\frac{\partial Loss}{\partial \mathbf{f}}\right|_{\mathbf{f}^0}' \mathbf{f} \tag{15}$$

[2]Under the assumption that the set $\mathcal{S}$ is non-islanding.

where for a line k,

$$\frac{\partial Loss}{\partial f_k} = \frac{2R_k}{V_k^2} f_k = x_k \tag{16}$$

Using (16) we can also express $Loss$ as

$$Loss = \frac{1}{2}\frac{\partial Loss}{\partial \mathbf{f}}'\mathbf{f} \tag{17}$$

Equating (17) and (15) for $\mathbf{f} = \mathbf{f}^0$ we can derive the bias term $\mathbf{b}^0$ as

$$Loss^0 = \frac{1}{2}\boldsymbol{x}^{0\prime}\mathbf{f}^0 = \mathbf{b}^0 + \boldsymbol{x}^{0\prime}\mathbf{f}^0 \rightarrow$$
$$\mathbf{b}^0 = -\frac{1}{2}\boldsymbol{x}^{0\prime}\mathbf{f}^0$$

Therefore, for any flow vector $\mathbf{f}$, we write losses as

$$Loss = \boldsymbol{x}^{0\prime}(\mathbf{f} - \frac{1}{2}\mathbf{f}^0) \tag{18}$$

The term $\boldsymbol{x}^0$ is referred to as the vector of line loss factors. The loss formulation we present here is similar to one used in real markets (e.g. [18]) where losses are included in the energy balance constraint and in the flow constraints via a nodal allocation of Losses (represented below by the normalized vector $\mathbf{d}$). Litvinov et al. [19] showed that the advantage of this formulation compared to other approaches is that line losses and flows are reference bus independent. The formulation below is slightly different from the one described in [19] and therefore, we will repeat the proof of reference bus independence in the Appendix. Without loss of generality, contingency constraints are excluded.[3]

$$\min_{\mathbf{p}} \mathbf{c}'\mathbf{p} \tag{19}$$
$$\text{s.t.} \quad 1'(\mathbf{p}-\mathbf{l}) = Loss \tag{20}$$
$$Loss = \boldsymbol{x}^{0\prime}\big(\mathbf{g}^0 + \mathbf{\Psi}(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) - \frac{1}{2}f^0\big) \tag{21}$$
$$\underline{\mathbf{f}} \leq \mathbf{g}^0 + \mathbf{\Psi}(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) \leq \overline{\mathbf{f}} \tag{22}$$
$$\underline{\mathbf{p}} \leq \mathbf{p} \leq \overline{\mathbf{p}} \tag{23}$$

We will refer to constraints (19)-(22) as Formulation $L1$. In the flow constraints (22) the $\mathbf{d}$ vector allocates $Loss$ to busses. Without this term, losses would be balanced at the reference bus and thus the formulation would be reference bus dependent. There are many ways to select $\mathbf{d}$ and we will not delve into this problem here. An intuitive approach, and the one we assume in this paper, is to set

$$d_n = \frac{l_n}{\sum_m l_m} \quad \forall n,$$

which allocates losses to load busses in proportion to their contribution to total load.

[3]Note that only losses in the base topology are included in the energy balance equation, losses in contingent topologies only impact contingent flows.

IV. TC MIP Formulation with Losses

As we saw in section II, FCTs linearly impact flows in the same way as updating the shift factor matrix. With the introduction of losses, however, these FCTs would no longer be balanced since the injection at one end of the line is not equal to the withdrawal at the other end. In the case of losses we must redefine FCTs as the loss-adjusted canceling flows that need to be introduced so that the effect from these flows is the same as actually opening the lines[4]. Fortunately, we can still retain the same framework of the lossless shift factor TC problem. To do this, we first derive the loss-adjusted shift factor matrix, $\hat{\mathbf{\Psi}}$ and loss-adjusted PTDF matrix, $\hat{\mathbf{\Phi}}$, by explicitly expressing flows in terms of losses.
Re-arranging (21) we have

$$Loss = \frac{\boldsymbol{x}^{0\prime}\big(\mathbf{g}^0 + \mathbf{\Psi}(\mathbf{p}-\mathbf{l}) - \frac{\mathbf{f}^0}{2}\big)}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1}$$

The flow equation can thus be expressed as

$$\begin{aligned}
\mathbf{f} &= \mathbf{g}^0 + \mathbf{\Psi}\Big(\mathbf{p}-\mathbf{l}-\mathbf{d}\frac{\boldsymbol{x}^{0\prime}\big(\mathbf{g}^0+\mathbf{\Psi}(\mathbf{p}-\mathbf{l})-\frac{\mathbf{f}^0}{2}\big)}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1}\Big) \\
&= \mathbf{g}^0 - \mathbf{\Psi}\mathbf{d}\frac{\boldsymbol{x}^{0\prime}(\mathbf{g}^0-\frac{\mathbf{f}^0}{2})}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1} + \mathbf{\Psi}(\mathbf{p}-\mathbf{l}) \\
&- \mathbf{\Psi}\mathbf{d}\frac{\boldsymbol{x}^{0\prime}\mathbf{\Psi}(\mathbf{p}-\mathbf{l})}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1} \\
&= \hat{\mathbf{g}}^0 + \mathbf{\Psi}\big(\mathbf{I}-\frac{\mathbf{d}\boldsymbol{x}^{0\prime}\mathbf{\Psi}}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1}\big)(\mathbf{p}-\mathbf{l}) \rightarrow \\
\mathbf{f} &= \hat{\mathbf{g}}^0 + \hat{\mathbf{\Psi}}(\mathbf{p}-\mathbf{l})
\end{aligned} \tag{24}$$

from which we see that the loss-adjusted shift factor matrix and flow bias are

$$\begin{aligned}
\hat{\mathbf{\Psi}} &= \mathbf{\Psi}\big(\mathbf{I}-\frac{\mathbf{d}\boldsymbol{x}^{0\prime}\mathbf{\Psi}}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1}\big) \\
\hat{\mathbf{g}}^0 &= \mathbf{g}^0 - \mathbf{\Psi}\mathbf{d}\frac{\boldsymbol{x}^{0\prime}(\mathbf{g}^0-\frac{\mathbf{f}^0}{2})}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1}
\end{aligned}$$

For completness the flow bias $\mathbf{g}^0$ can be calculated as:

$$\begin{aligned}
\mathbf{g}^0 &= \mathbf{f^0} - \mathbf{\Psi}(\mathbf{p}^0 - \mathbf{l}^0 - \mathbf{d}\cdot Loss^0) \\
&= \mathbf{f^0} - \mathbf{\Psi}(\mathbf{p}^0 - \mathbf{l}^0 - \mathbf{d}\boldsymbol{x}^{0\prime}\frac{\mathbf{f^0}}{2}) \\
&= (I + \mathbf{\Psi}\mathbf{d}\frac{\boldsymbol{x}^{0\prime}}{2})\mathbf{f^0} - \mathbf{\Psi}(\mathbf{p}^0 - \mathbf{l}^0)
\end{aligned}$$

The loss-adjusted PTDF can now be expressed as:

$$\begin{aligned}
\hat{\phi}^\ell_k &= \hat{\Psi}^{m_\ell}_k - \hat{\Psi}^{n_\ell}_k \\
&= \big(\psi^{m_\ell}_k - \boldsymbol{\psi}^{m_\ell\prime}\frac{\boldsymbol{x}^0(\boldsymbol{\psi}_k\mathbf{d})}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1}\big) \\
&- \big(\psi^{n_\ell}_k - \boldsymbol{\psi}^{n_\ell\prime}\frac{\boldsymbol{x}^0(\boldsymbol{\psi}_k\mathbf{d})}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1}\big) \\
&= \phi^\ell_k - \boldsymbol{\phi}^{k\prime}\boldsymbol{x}^0\frac{\boldsymbol{\psi}_k\mathbf{d}}{\boldsymbol{x}^{0\prime}\mathbf{\Psi}\mathbf{d}+1}
\end{aligned}$$

where $\boldsymbol{\psi}^{m_\ell}$ denotes the column of $\mathbf{\Psi}$ corresponding to bus m and $\boldsymbol{\phi}^k$ denotes the column of $\mathbf{\Phi}$ corresponding to line k. Although this derivation is not necessary for the MIP formulation it provides some intuition for how the PTDF is adjusted for losses. We should note that although the loss-adjusted shift factors and PTDF matrices depend on an initial dispatch and set of flows, they can neverthless be pre-calculated from the original shift factor matrix using only matrix multiplication, which would be fast even for large systems.
With these loss-adjusted shift factors and PTDFs we can now express loss-adjusted FCTs similarly to (13). In addition, since the $Loss$ equation is a function of flows, we apply FCTs to update $Loss$ for line openings. Partitioning transmission losses into losses from monitored and switchable lines respectively, we have:

$$\begin{aligned}
Loss^{\mathcal{M}} &= \boldsymbol{x}^{0\prime}\big(\mathbf{g}^{0\mathcal{M}} - \frac{1}{2}\mathbf{f}^{0\mathcal{M}} + \mathbf{\Psi}^{\mathcal{M}}_{\tau_0}(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) \\
&+ \hat{\mathbf{\Phi}}^{\mathcal{MS}}_{\tau_0}\mathbf{v}_{\tau_0}\big) \\
Loss^{\mathcal{S}} &= \boldsymbol{x}^{0\prime}\big(\mathbf{g}^{0\mathcal{S}} - \frac{1}{2}\mathbf{f}^{0\mathcal{S}} + \mathbf{\Psi}^{\mathcal{S}}_{\tau_0}(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) \\
&+ (I - \hat{\mathbf{\Phi}}^{\mathcal{SS}}_{\tau_0})\mathbf{v}_{\tau_0}\big)
\end{aligned}$$

Finally, the full topology control DC SCOPF MIP with losses is:

$$\mathcal{C} = \min_{\mathbf{p},\mathbf{v},\mathbf{z}} \mathbf{c}'\mathbf{p} \tag{25}$$

$$\text{s.t.}\quad \mathbf{1}'(\mathbf{p}-\mathbf{l}) - Loss = 0 \tag{26}$$

$$Loss = Loss^{\mathcal{M}} + Loss^{\mathcal{S}} \tag{27}$$

$$Loss^{\mathcal{M}} = \boldsymbol{x}^{0\mathcal{M}\prime}\big(\hat{\mathbf{g}}^{0\mathcal{M}} - \frac{1}{2}f^{0\mathcal{M}} + \hat{\mathbf{\Psi}}^{\mathcal{M}}_{\tau_0}(\mathbf{p}-\mathbf{l}) + \hat{\mathbf{\Phi}}^{\mathcal{MS}}_{\tau_0}\mathbf{v}_{\tau_0}\big) \tag{28}$$

$$Loss^{\mathcal{S}} = \boldsymbol{x}^{0\mathcal{S}\prime}\big(\hat{\mathbf{g}}^{0\mathcal{S}} - \frac{1}{2}f^{0\mathcal{S}} + \hat{\mathbf{\Psi}}^{\mathcal{S}}_{\tau_0}(\mathbf{p}-\mathbf{l}) + (\hat{\mathbf{\Phi}}^{\mathcal{SS}}_{\tau_0} - \mathbf{I})\mathbf{v}_{\tau_0}\big) \tag{29}$$

$$\underline{\mathbf{f}}^{\mathcal{M}}_\tau \le \hat{\mathbf{g}}^0_\tau + \hat{\mathbf{\Psi}}^{\mathcal{M}}_\tau(\mathbf{p}-\mathbf{l}) + \hat{\mathbf{\Phi}}^{\mathcal{MS}}_\tau\mathbf{v}_\tau \le \overline{\mathbf{f}}^{\mathcal{M}}_\tau, \quad \forall\tau \tag{30}$$

$$\underline{\tilde{\mathbf{F}}}^{\mathcal{S}}_\tau\mathbf{z} \le \hat{\mathbf{g}}^0_\tau + \hat{\mathbf{\Psi}}^{\mathcal{S}}_\tau(\mathbf{p}-\mathbf{l}) + \big(\hat{\mathbf{\Phi}}^{\mathcal{SS}}_\tau - \mathbf{I}\big)\mathbf{v}_\tau \le \overline{\tilde{\mathbf{F}}}^{\mathcal{S}}_\tau\mathbf{z}, \quad \forall\tau \tag{31}$$

$$-M(\mathbf{1}-\mathbf{z}) \le \mathbf{v}_\tau \le M(\mathbf{1}-\mathbf{z}), \quad \forall\tau \tag{32}$$

$$\underline{\mathbf{p}} \le \mathbf{p} \le \overline{\mathbf{p}} \tag{33}$$

$$z_\ell \in \{0,1\}, \quad \forall\ell\in\mathcal{S} \tag{34}$$

We refer to problem (25)-(34) as the loss-adjusted shift factor TC formulation. Note that the $\mathbf{d}\cdot Loss$ term in the above formulation is replaced via the equivalence relationship shown in (24). Generally, ISOs may only monitor a subset of all transmission lines for losses. This means that $Loss$ will be aggregated over a set that is smaller than $\mathcal{M}\cup\mathcal{S}$. In the extreme case when no lines in $\mathcal{S}$ are monitored for losses, constraint (29) would be empty. If additionally the end nodes of lines in $\mathcal{S}$ are not load busses ($d_m, d_n = 0$), constraints (31) would reduce to (10) and FCTs would be calculated independent of losses.

V. Locational Marginal Prices and Losses

In this section we derive LMPs under the loss-adjusted shift factor TC formulation. By definition the LMP of problem (25)-(34) equals the derivative of the Lagrangian with respect to a

[4]These FCTs will be adjusted to account for losses across all transmission lines, measured at the receiving ends.

change in nodal load. Let the Lagrangian multipliers or shadow prices associated with constraints (26), combined constraints (27)-(29), (30) and (31) be denoted by λ, η, $\underline{\boldsymbol{\mu}}$ and $\overline{\boldsymbol{\mu}}$, and $\underline{\boldsymbol{\alpha}}$ and $\overline{\boldsymbol{\alpha}}$ respectively, where the latter four shadow prices are the collection of the respective contingent topology shadow prices. Using these shadow prices, LMPs are given by

$$\begin{aligned} LMP &= \lambda\mathbf{1} - \lambda(\hat{\boldsymbol{\Psi}}^{\mathcal{M}'}_{\tau_0}\boldsymbol{x}^{0^{\mathcal{M}}} + \hat{\boldsymbol{\Psi}}^{\mathcal{S}'}_{\tau_0}\boldsymbol{x}^{0^{\mathcal{S}}}) \\ &- \hat{\boldsymbol{\Psi}}^{\mathcal{M}'}(\overline{\boldsymbol{\mu}} - \underline{\boldsymbol{\mu}}) - \hat{\boldsymbol{\Psi}}^{\mathcal{S}'}(\overline{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}}) = \\ &= \lambda\mathbf{1} - \lambda\hat{\boldsymbol{\Psi}}'_{\tau_0}\boldsymbol{x}^0 \\ &- \hat{\boldsymbol{\Psi}}^{\mathcal{M}'}(\overline{\boldsymbol{\mu}} - \underline{\boldsymbol{\mu}}) - \hat{\boldsymbol{\Psi}}^{\mathcal{S}'}(\overline{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}}) \end{aligned} \quad (35)$$

where $\hat{\boldsymbol{\Psi}}^{\mathcal{M}}$ and $\hat{\boldsymbol{\Psi}}^{\mathcal{S}}$ are matrices that consist of the collection of $\hat{\boldsymbol{\Psi}}^{\mathcal{M}}_{\tau}$ and $\hat{\boldsymbol{\Psi}}^{\mathcal{S}}_{\tau}$, for all contingent topologies τ.

To better understand the meaning of LMPs under this MIP formulation we consider two alternate non-MIP formulations [20] where we fix $\mathbf{z} = \mathbf{z}^*$(optimal values from the MIP solution). In the first formulation, which we refer to as the Static MIP problem, we re-label all lines with $z = 1$ as belonging to monitored set $\mathcal{M}$[5] and resolve problem (25)-(34). The LMPs for the Static MIP problem are of the same form as (35). In the second formulation, which we refer to as the LP-Equivalent, we recalculate the shift factor matrix given z^* and solve a standard SCOPF problem where the base topology has the lines with $z = 0$ removed. The shift factor matrix given z^* is expressed as [16]

$$\hat{\boldsymbol{\Psi}}^{\mathcal{M}*} = \hat{\boldsymbol{\Psi}}^{\mathcal{M}} + \hat{\mathbf{O}}^{\mathcal{M}\mathcal{S}}\hat{\boldsymbol{\Psi}}^{\mathcal{S}}, \quad (36)$$

where $\hat{\mathbf{O}}^{\mathcal{M}\mathcal{S}}$ is the loss-adjusted line outage distribution factor matrix indicating the impact of switched lines on monitored lines for each contingent topology. The LMPs in the LP-Equivalent problem are defined in the standard manner as

$$LMP^* = \lambda\mathbf{1} - \lambda\hat{\boldsymbol{\Psi}}^{\mathcal{M}*'}_{\tau_0}\boldsymbol{x}^{0^{\mathcal{M}}} - \hat{\boldsymbol{\Psi}}^{\mathcal{M}*'}(\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*). \quad (37)$$

With $\mathbf{z} = \mathbf{z}^*$, the solution to the Static MIP and LP-Equivalent problems are identical[6] and, therefore, the LMPs and shadow prices associated with flow limits on transmission elements (given our relabeling of lines in the Static MIP problem) must also be identical.

$$\begin{aligned} LMP &= LMP^* \quad (38)\\ \boldsymbol{\mu} &= \boldsymbol{\mu}^*. \quad (39) \end{aligned}$$

Substituting (35) and (37) into (38) and canceling the energy component yields,

$$\begin{aligned} &\lambda(\hat{\boldsymbol{\Psi}}^{\mathcal{M}'}_{\tau_0}\boldsymbol{x}^{0^{\mathcal{M}}} + \hat{\boldsymbol{\Psi}}^{\mathcal{S}'}_{\tau_0}\boldsymbol{x}^{0^{\mathcal{S}}}) + \hat{\boldsymbol{\Psi}}^{\mathcal{M}'}(\overline{\boldsymbol{\mu}} - \underline{\boldsymbol{\mu}}) + \hat{\boldsymbol{\Psi}}^{\mathcal{S}'}(\overline{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}}) = \\ &\lambda\hat{\boldsymbol{\Psi}}^{\mathcal{M}*'}_{\tau_0}\boldsymbol{x}^{0^{\mathcal{M}}} + \hat{\boldsymbol{\Psi}}^{\mathcal{M}'*}(\overline{\boldsymbol{\mu}}^* - \underline{\boldsymbol{\mu}}^*). \end{aligned} \quad (40)$$

Furthermore, substituting (36) and (39) into (40) and appropriately canceling like terms yields,

$$\begin{aligned} \hat{\boldsymbol{\Psi}}^{\mathcal{S}'}(\overline{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}}) &= \lambda\hat{\boldsymbol{\Psi}}^{\mathcal{S}'}_{\tau_0}\hat{\mathbf{O}}^{\mathcal{M}\mathcal{S}'}_{\tau_0}\boldsymbol{x}^{0^{\mathcal{M}}} - \lambda\hat{\boldsymbol{\Psi}}^{\mathcal{S}'}_{\tau_0}\boldsymbol{x}^{0^{\mathcal{S}}} \\ &+ \hat{\boldsymbol{\Psi}}^{\mathcal{S}'}\hat{\mathbf{O}}^{\mathcal{M}\mathcal{S}'}(\overline{\boldsymbol{\mu}} - \underline{\boldsymbol{\mu}}). \end{aligned} \quad (41)$$

[5]As mentioned previously, for all lines ℓ with $z_\ell = 1$, $v_\ell = 0$ for all topologies and constraints (31) reduce to constraints (30).

[6]We do not recalculate line loss factors in the LP-Equivalent problem as this would yield a different solution.

Finally, by substituting (41) into (35), canceling like terms and grouping appropriately, we see that the LMP derived from the loss-adjusted TC MIP formulation is

$$\begin{aligned} LMP &= \lambda\mathbf{1} - \lambda\Big(\hat{\boldsymbol{\Psi}}^{\mathcal{M}'}_{\tau_0} + \hat{\boldsymbol{\Psi}}^{\mathcal{S}'}_{\tau_0}\hat{\mathbf{O}}^{\mathcal{M}\mathcal{S}'}_{\tau_0}\Big)\boldsymbol{x}^{0^{\mathcal{M}}} \\ &- \Big(\hat{\boldsymbol{\Psi}}^{\mathcal{M}'} + \hat{\boldsymbol{\Psi}}^{\mathcal{S}'}_{\tau_0}\hat{\mathbf{O}}^{\mathcal{M}\mathcal{S}'}\Big)(\overline{\boldsymbol{\mu}} - \underline{\boldsymbol{\mu}}) \end{aligned} \quad (42)$$

As an aside, we observe that equation (41) has a loss component, which is only relevant in topology τ_0. By appropriately grouping terms by topology we can partition (41) into two components:

$$\begin{aligned} (\overline{\boldsymbol{\alpha}}_{\tau_0} - \underline{\boldsymbol{\alpha}}_{\tau_0}) &= \hat{\mathbf{O}}^{\mathcal{M}\mathcal{S}'}_{\tau_0}(\overline{\boldsymbol{\mu}}_{\tau_0} - \underline{\boldsymbol{\mu}}_{\tau_0}) \\ &+ \lambda\Big(\hat{\mathbf{O}}^{\mathcal{M}\mathcal{S}'}_{\tau_0}\boldsymbol{x}^{0^{\mathcal{M}}} - \boldsymbol{x}^{0^{\mathcal{S}}}\Big) \end{aligned} \quad (43)$$

and

$$(\overline{\boldsymbol{\alpha}}_{\tau} - \underline{\boldsymbol{\alpha}}_{\tau}) = \hat{\mathbf{O}}^{\mathcal{M}\mathcal{S}'}_{\tau}(\overline{\boldsymbol{\mu}}_{\tau} - \underline{\boldsymbol{\mu}}_{\tau}) \quad \forall\tau \neq \tau_0 \quad (44)$$

Expressions (43)-(44) are similar to the relationship between shadow prices for switchable and monitored lines reported in [20]. This relationship provides a generalization of the "total derivative" concept for line openings introduced in [14], where $\overline{\boldsymbol{\alpha}}_{\tau} - \overline{\boldsymbol{\alpha}}_{\tau}$ reflects the marginal value (positive or negative) of line switching. As shown in (43), this value consists of both congestion and loss components. The congestion component is the scalar product of shadow prices for monitored constraints and LODFs of the open line on these constraints. The loss component is the difference between: the impact of line opening on losses in monitored facilities (LODFs multiplied by the corresponding line loss factors) and, the loss factors of open lines. While in the lossless formulation topology change is never beneficial in the absence of transmission congestion (congestion component equals zero), incorporating marginal losses recognizes potential benefits of topology control due to a reduction in losses, which may be realized in the absence of congestion.

VI. Simulation Results

For all analyses we fixed a set of 16 switchable lines (to maintain tractable run times as mentioned in the introduction) and performed a Monte Carlo simulation by randomly varying fuel prices and wind capacity over 100 samples[7]. For each sample we modeled the lossless DC SCOPF and the DC SCOPF with shift factors and bias terms adjusted for losses[8]. Taking the optimal topology from each of the two formulations we solved the AC SCOPF for each sample to assess the feasibility of the DC solutions and the (p.u.) congestion cost savings[9]. Congestion cost savings for the DC and AC models are calculated relative to the DC and AC models with no TC respectively. Table I below summarizes the results from

[7]See [14] for details on this approach.

[8]In the lossless formulation the bias term is calculated as $\mathbf{g}^0 = \mathbf{f^0} - \boldsymbol{\Psi}(\mathbf{p}^0 - \mathbf{l}^0)$.

[9]Per-unit (p.u.) congestion savings are calculated as $\frac{C_{MIP} - C_{base}}{C_{base} - C_0}$ where C_{MIP} is the system cost from the DC MIP or from the AC OPF based on this MIP, C_{base} is the DC or AC system costs with no switching and C_0 is the DC or AC system cost with no enforced transmission constraints. $C_{base} - C_0$ represents the maximum savings possible for any sample.

TABLE I
MEDIAN NUMBER OF LINE OPENINGS AND AVERAGE PER-UNIT CONGESTION COSTS SAVINGS WITH DIFFERENT LOSS MODELING ASSUMPTIONS OVER 100 SAMPLES

Model	Num. Lines Opened	Congestion Cost Savings with TC
DC w/Losses	11	16.64%
DC Lossless	14	22.09%
AC Based on DC w/Losses	N/A	20.66%
AC Based on DC Lossless	N/A	20.65%

which we make three key observations. First, based on the AC SCOPF solutions, accounting for losses does not improve congestion cost savings. The AC OPF based on the loss-adjusted DC MIP solution produces more savings in 53 of the samples, compared to the AC OPF based on the lossless DC MIP, however, the magnitude in savings tends to be greater for the latter model so that both AC solutions lead to about the same 20.65% p.u. congestion cost savings. Second, solving the AC SCOPF using the optimal topologies from the two DC formulations confirms that both DC formulations reflect legitimate congestion cost savings. The lossless MIP formulation overstates the benefits from switching by an average of 1.44% in 76 samples while the loss-adjusted MIP understates them by 4.03% in 93 samples. Both formulation, however, provide a good indication of potential savings from topology control. Finally, we observe that the loss-adjusted DC MIP tends to open 3 fewer lines than its lossless counterpart. Opening fewer lines is clearly preferable; operating circuit breakers, although not included explicitly modeled, has a cost and, achieving the same savings with fewer openings is more efficient and reduces the number of discrete changes to the state of the system. Figure 1 below shows the number of line openings across all samples. As shown in the figure, the loss-adjusted

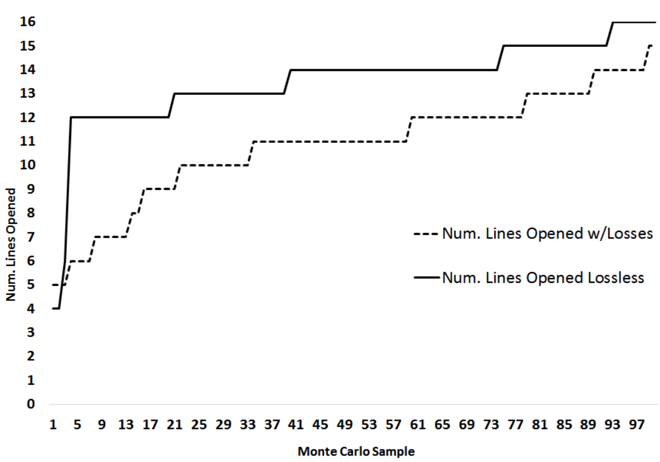

Fig. 1. Number of Lines Opened in DC MIP Formulations

MIP opens between 1 and 9 fewer lines in 79 samples, opens the same number of lines in 18 samples, and opens only 1 additional line in 3 samples.

VII. CONCLUSION

In this paper, we develop a loss-adjusted MIP-based TC formulation and show that both losses and flows can be updated linearly with changes in topology. We prove that our formulation is reference bus independent and extend the notion of FCTs to account for losses by deriving loss-adjusted shift factors and PTDFs. We derive locational marginal prices (LMPs) for the LP equivalent to the loss-adjusted TC MIP formulation and show how the marginal value of transmission switching can be expressed in terms of a congestion and loss component. Through simulation, we analyze the impact of losses on the DC formulation and compare the optimal topologies from the lossless and loss-adjusted formulations by solving an AC SCOPF. We find that both DC formulations lead to almost identical savings when solving the AC SCOPF and while the loss-adjusted formulation opens fewer lines both can be used reliably to assess the benefits of topology control. Future work will focus on adjusting loss factors not only for topology changes but also for re-dispatch. Although this would make the problem non-linear, it may lead to a better solution in terms of the number of line openings, as indicated by our results, or congestion cost savings. One approach for addressing the non-linearity is to iterate on the MIP solution, updating loss factors after each iteration, although the practicality of this approach along with other alternatives needs to be studied.

APPENDIX

Proof that formulation $L1$ is reference bus independent

To demonstrate that $L1$ is reference bus independent we modify the choice of reference bus by introducing a normalized vector $\mathbf{w}$ that assigns a weighting to each node in proportion to its contribution to the new distributed reference bus (a single reference bus n can be represented by setting $w_n = 1$). As shown in [19], a weighting $\mathbf{w}$ modifies the shift factor matrix according to

$$\mathbf{\Psi_w} = \mathbf{\Psi} - \mathbf{\Psi w 1'} \tag{45}$$

Substituting (45) into (21) gives

$$\begin{aligned} Loss_{\mathbf{w}} &= \boldsymbol{x}^{0\prime}\big(\mathbf{g}^0 + (\mathbf{\Psi} - \mathbf{\Psi w 1'})\cdot \\ &\quad (\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) - \frac{1}{2}f^0\big) = \\ &\boldsymbol{x}^{0\prime}\big(\mathbf{g}^0 + \mathbf{\Psi}(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) - \frac{1}{2}f^0\big) - \\ &\boldsymbol{x}^{0\prime}\mathbf{\Psi w 1'}(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) = Loss \end{aligned}$$

since $\mathbf{1'}(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) = 0$. Similarly, substituting (45) into (22) gives

$$\begin{aligned} \mathbf{f_w} = \mathbf{g}^0 + (\mathbf{\Psi} - \mathbf{\Psi w 1'})(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) &= \\ \mathbf{g}^0 + \mathbf{\Psi}(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) &- \\ \mathbf{\Psi w 1'}(\mathbf{p}-\mathbf{l}-\mathbf{d}\cdot Loss) &= \mathbf{f} \end{aligned}$$

We have thus shown that the constraint set is reference bus independent. Further, since the objective function is reference bus independent, by definition, the shadow prices will be reference bus independent as well.

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